Geometry, Quarter 4, Unit 4.1

Arc Length and Equations of Circles and Parabolas

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Know and use definitions in geometric constructions.
- Derive the length of an arc from its intercepted central angle or inscribed angle and its radius.
- Define radian measure.
- Derive the formula for the area of a sector of a circle.
- Explore secant and tangent lines and the properties of central, inscribed, and circumscribed angles.
- Explore the properties of tangent lines at the point of intersection of a circle and its radius.
- Explore the relationship between the center and radius of a circle given by an equation.
- Explore the relationship between the equation of a circle and its center and radius.
- Explore the relationship between the equation of a parabola and its focus and directrix.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Graph circles to find arc lengths and measures.
- Use constructions with lines and angles in a circle.

Model with mathematics.

- Use a graphing calculator to graph secants and tangents within a circle.
- Use a protractor and ruler to construct circles.
- Use graph paper to show precise coordinate points on a circle.

Essential questions

- What methods can be used to determine arc lengths and measures?
- What are the similarities and differences between inscribed and circumscribed angles in a circle?
- How do you determine the measure of the arc of a circle that is intercepted by an angle, formed by secant/tangent that is proportional to the radius?
- What is the relationship between the equation of a circle and its radius and center?
- How do you find the center and radius of a circle given by an equation?
- What is the equation of a parabola given the focus and directrix?

Written Curriculum

Common Core State Standards for Mathematical Content

Congruence G-CO

Experiment with transformations in the plane

G-CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Circles G-C

Find arc lengths and areas of sectors of circles

G-C.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Expressing Geometric Properties with Equations G-GPE

Translate between the geometric description and the equation for a conic section

G-GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

G-GPE.2 Derive the equation of a parabola given a focus and directrix.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

In Kindergarten, students identified and described shapes. In grades 1 and 3, students reasoned with shapes and their attributes. In grade 4, students drew and identified lines and angles and classified shapes by properties of their lines and angles. In grade 5, students classified two-dimensional figures into categories based on their properties. In grade 7, students drew, constructed, and described geometrical figures, and they described the relationships between the figures. In grade 8, students understood and applied the Pythagorean Theorem. In Algebra 1, students created equations that described numbers or relationships. They also understood that solving equations was a process of reasoning and explained the reasoning, solved equations and inequalities in one variable, solved systems of equations, and represented and solved equations and inequalities graphically.

Current Learning

Students derive and interpret the length of an arc that is proportional to the radius. Also, they define the radian measure of angles. Students derive formulas for the area of a sector of a circle. In this unit, students explore the relationship between the center and radius of a circle. They explore the relationship between the equation of a parabola and its focus and directrix. Students know and use definitions in geometric constructions.

Future Learning

In Unit 4.2 of this course, students will use the knowledge gained in explorations to derive equations of circles and parabolas and to derive the formula for the area of a sector. In Algebra 2, students will use a coordinate plane and trigonometry to model periodic phenomena. They will also use their experience in Geometry to create models and solve contextual problems. Students will also use parabola, special right triangles, the unit circle, and trigonometric functions to solve problems.

Additional Findings

None found.

Geometry, Quarter 4, Unit 4.2

Investigating Equations of Circles and Parabolas

Overview

Number of instructional days: 13 (1 day = 45–60 minutes)

Content to be learned

- Use the Pythagorean Theorem to derive the equation of a circle.
- Complete the square of an equation to find the center and the radius of a circle.
- Derive the equation of a parabola given the focus and directrix.

Mathematical practices to be integrated

Model with mathematics.

- Use graph paper to determine the focus and directrix of the parabola.
- Use the Pythagorean Theorem and diagrams on the coordinate plane to derive the equation of a circle.

Use appropriate tools strategically

- Use a graphing calculator to graph circles.
- Use a compass and ruler to graph parabolas and circles.
- Use graph paper to graph equations of parabolas.
- Use dynamic software to view circles and parabolas.

Attend to precision.

- Use precise mathematical language to define the focus of a parabola.

Look for and make use of structure.

- Use the Pythagorean Theorem to find the center of circles and parabolas.
- Complete the square to determine the structure of the equation of a circle and to reveal the center and radius.

Essential questions

- How do you use the Pythagorean Theorem to derive the equation of a circle?
- What methods can be used to complete the square of an equation to find the center and radius of a circle?
- How do you derive the equation of a parabola given the focus and directrix?

Written Curriculum

Common Core State Standards for Mathematical Content

Expressing Geometric Properties with Equations	G-GPE

Translate between the geometric description and the equation for a conic section

G-GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

G-GPE.2 Derive the equation of a parabola given a focus and directrix.

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In Kindergarten, students identified and described shapes, and in grade 1 they reasoned with shapes and their attributes. In grade 4, students drew and identified lines and angles, and they classified shapes by properties of their lines and angles. In grade 5, students classified two-dimensional figures into categories based on their properties. In grade 7, students drew, constructed, and described geometrical figures, and they described the relationships between them. In grade 8, students understood and applied the Pythagorean Theorem, and in Algebra 1, they understood solving equations as a process of reasoning and explaining the reasoning. They also solved equations and inequalities in one variable and solved systems of equations.

Current Learning

In this unit, students use the Pythagorean Theorem to derive the equation of a circle, and they will use the distance formula to write equations of a circle when given the radius and the coordinates of a center. Students use the equation of a circle to find segment lengths of secants and tangents. They complete the square of an equation to find the center and the radius of a circle, and they derive the equation of a parabola given the focus and directrix. They also graph on a coordinate plane the quadratic equation for the parabola.

Future Learning

Students' previous work in geometry regarding trigonometric ratios and circles can now expand the use of the coordinate plane to model periodic phenomena. They also expand their work using exponential functions to include solving exponential equations with logarithms.

Additional Findings

In this unit, students expand their knowledge of circle parts to derive equations of circles. Given the equation of a circle, students draw the graph in a coordinate plane and apply techniques for solving quadratic equations.

Geometry, Quarter 4, Unit 4.3

Exploring the Laws of Sines and Cosines

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Derive the formula A = ½ ab sin(C) for the area of a triangle.
- Prove the Laws of Sines and Cosines.
- Explore the Laws of Sines and Cosines to understand them and compare them.
- Experiment with two-dimensional figures in relation to sine and cosine to deepen understanding of the Laws of Sines and Cosines.
- Apply the Laws of Sines and Cosines to find missing measures of general triangles.
- Use the Laws of Sines and Cosines to solve real-world problems.

Mathematical practices to be integrated

Model with mathematics.

- Use a graphing calculator to explore triangles by using the Laws of Sines and Cosines.
- Use constructions to explore and define the Laws of Sines and Cosines.

Look for and make use of structure.

- Examine the structure of a triangle that is not a right triangle to define the Laws of Sines and Cosines.

Look for and express regularity in repeated reasoning.

- Use definitions to prove the Laws of Sines and Cosines.

Essential questions

- How do you derive the formula A = ½ ab sin (C)?
- How do the Laws of Sines and Cosines compare?
- How do you prove the Laws of Sines and Cosines?
- How do you determine when to use the Laws of Sines or the Law of Cosines?
- How can trigonometry be used on triangles that are not right triangles?
- Why can't the Law of Sines be used to determine the measures of the missing angles and/or sides?

Written Curriculum

Common Core State Standards for Mathematical Content

Similarity, Right Triangles, and Trigonometry	G-SRT

Apply trigonometry to general triangles

G-SRT.9 (+) Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

G-SRT.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems.

G-SRT.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In grade 1, students composed two-dimensional shapes such as triangles, rectangles, and squares. In grade 4, students recognized right triangles as a category, and they identified right triangles. In grade 6, students computed the area of right triangles and other triangles. In grade 8, students explained and used a proof of the Pythagorean Theorem and its converse.

Current Learning

Students derive the formula for finding the area of a triangle. They understand and prove the Laws of Sines and Cosines, and they use those laws to solve problems, including finding missing measures of general triangles, not always right triangles. Students also apply similarity in right triangles to understand right triangle trigonometry, especially right triangles and the Pythagorean Theorem.

Future Learning

In Algebra 2, students expand their knowledge of trigonometric functions to prove addition and subtraction formulas for sine, cosine, and tangent.

Additional Findings

In this unit, student will apply trigonometry to general triangles, where the definitions of sine and cosine must be extended to obtuse angles. They will also focus on situations that require relating to two- and three-dimensional objects using the Laws of Sines and Cosines.